C3 Trigonometry

1. June 2010 qu.3

(i) Express the equation cosec $\theta(3\cos 2\theta + 7) + 11 = 0$ in the form $a \sin^2\theta + b\sin\theta + c = 0$, where a, b and c are constants. [3]

(ii) Hence solve, for $-180° < \theta < 180°$, the equation cosec $\theta(3\cos 2\theta + 7) + 11 = 0$. [3]

2. June 2010 qu.8

(i) Express $3\cos x + 3\sin x$ in the form $R\cos(x - \alpha)$, where $R > 0$ and $0 < \alpha < \frac{1}{2}\pi$. [3]

(ii) The expression T(x) is defined by $\mathrm{T}(x) = \dfrac{8}{3\cos x + 3\sin x}$.

(a) Determine a value of x for which T(x) is not defined. [2]

(b) Find the smallest positive value of x satisfying $\mathrm{T}(3x) = \frac{8}{9}\sqrt{6}$, giving your answer in an exact form. [4]

3. Jan 2010 qu.2

The angle θ is such that $0° < \theta < 90°$.

(i) Given that θ satisfies the equation $6\sin 2\theta = 5\cos\theta$, find the exact value of $\sin\theta$. [3]

(ii) Given instead that θ satisfies the equation $8\cos\theta\,\mathrm{cosec}^2\,\theta = 3$, find the exact value of $\cos\theta$. [5]

4. Jan2010 qu.9

The value of tan 10° is denoted by p. Find, in terms of p, the value of

(i) tan 55°, [3]

(ii) tan 5°, [4]

(iii) $\tan\theta$, where θ satisfies the equation $3\sin(\theta + 10°) = 7\cos(\theta - 10°)$. [5]

5. June 2009 qu.1

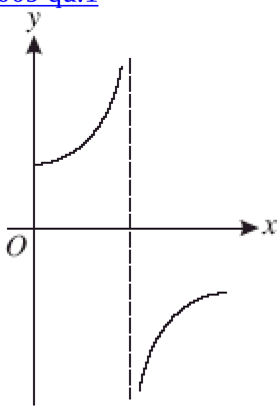


Fig. 1

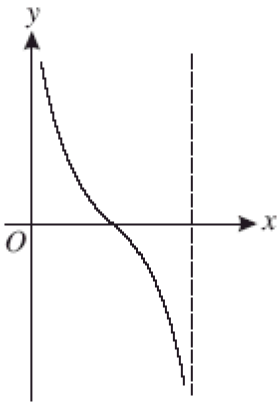


Fig. 2

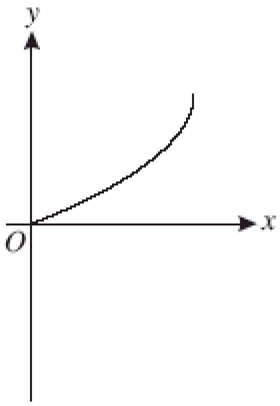


Fig. 3

Each diagram above shows part of a curve, the equation of which is one of the following:

$y = \sin^{-1} x$, $\quad y = \cos^{-1} x$, $\quad y = \tan^{-1} x$, $\quad y = \sec x$, $\quad y = \mathrm{cosec}\, x$, $\quad y = \cot x$.

State which equation corresponds to

(i) Fig. 1, [1]

(ii) Fig. 2, [1]

(iii) Fig. 3. [1]

6. June 2009 qu.3

The angles α and β are such that $\tan\alpha = m + 2$ and $\tan\beta = m$, where m is a constant.

(i) Given that $\sec^2\alpha - \sec^2\beta = 16$, find the value of m. [3]

(ii) Hence find the exact value of $\tan(\alpha + \beta)$. [3]

7. June 2009 qu.7

(i) Express $8 \sin \theta - 6 \cos \theta$ in the form $R \sin(\theta - \alpha)$, where $R > 0$ and $0° < \alpha < 90°$. [3]

(ii) Hence

(a) solve, for $0° < \theta < 360°$, the equation $8 \sin \theta - 6 \cos \theta = 9$, [4]

(b) find the greatest possible value of $32 \sin x - 24 \cos x - (16 \sin y - 12 \cos y)$ as the angles x and y vary. [3]

8. Jan 2009 qu.3

(i) Express $2 \tan^2 \theta - \dfrac{1}{\cos \theta}$ in terms of $\sec \theta$. [3]

(ii) Hence solve, for $0° < \theta < 360°$, the equation $2 \tan^2 \theta - \dfrac{1}{\cos \theta} = 4$. [4]

9. Jan 2009 qu.9

(i) By first expanding $\cos(2\theta + \theta)$, prove that $\cos 3\theta \equiv 4 \cos^3 \theta - 3 \cos \theta$. [4]

(ii) Hence prove that $\cos 6\theta \equiv 32 \cos^6 \theta - 48 \cos^4 \theta + 18 \cos^2 \theta - 1$. [3]

(iii) Show that the only solutions of the equation $1 + \cos 6\theta = 18 \cos^2 \theta$ are odd multiples of 90°. [5]

10. June 2008 qu.5

(a) Express $\tan 2\alpha$ in terms of $\tan \alpha$ and hence solve, for $0° < \alpha < 180°$, the equation

$$\tan 2\alpha \tan \alpha = 8.$$ [6]

(b) Given that β is the acute angle such that $\sin \beta = \dfrac{6}{7}$, find the exact value of

(i) $\operatorname{cosec} \beta$, [1]

(ii) $\cot^2 \beta$. [2]

11. June 2008 qu.8

The expression $T(\theta)$ is defined for θ in degrees by $T(\theta) = 3\cos(\theta - 60°) + 2\cos(\theta + 60°)$.

(i) Express $T(\theta)$ in the form $A \sin \theta + B \cos \theta$, giving the exact values of the constants A and B. [3]

(ii) Hence express $T(\theta)$ in the form $R \sin(\theta + \alpha)$, where $R > 0$ and $0° < \alpha < 90°$. [3]

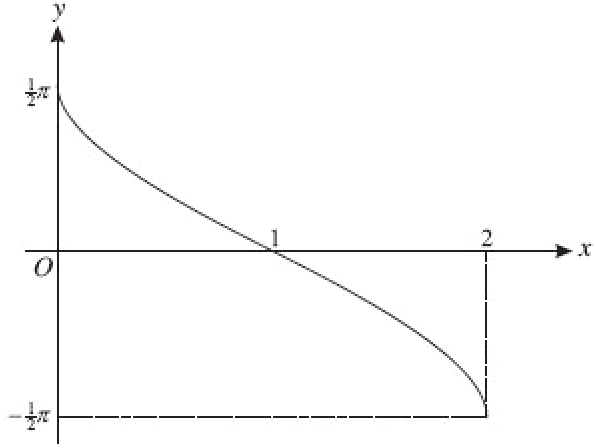
(iii) Find the smallest positive value of θ such that $T(\theta) + 1 = 0$. [4]

12. Jan 2008 qu.3

(a) Solve, for $0° < \alpha < 180°$, the equation $\sec \frac{1}{2}\alpha = 4$. [3]

(b) Solve, for $0° < \beta < 180°$, the equation $\tan \beta = 7 \cot \beta$. [4]

13. Jan 2008 qu.6



The diagram shows the graph of $y = -\sin^{-1}(x - 1)$.

(i) Give details of the pair of geometrical transformations which transforms the graph of $y = -\sin^{-1}(x - 1)$ to the graph of $y = \sin^{-1} x$. [3]

(ii) Sketch the graph of $y = \left|-\sin^{-1}(x-1)\right|$. [2]

(iii) Find the exact solutions of the equation $\left|-\sin^{-1}(x-1)\right| = \frac{1}{3}\pi$. [3]

14. Jan 2008 qu.9

(i) Use the identity for $\cos(A + B)$ to prove that $4\cos(\theta + 60°)\cos(\theta + 30°) \equiv \sqrt{3} - 2\sin 2\theta$. [4]

(ii) Hence find the exact value of $4\cos 82.5° \cos 52.5°$. [2]

(iii) Solve, for $0° < \theta < 90°$, the equation $4\cos(\theta + 60°)\cos(\theta + 30°) = 1$. [3]

(iv) Given that there are no values of θ which satisfy the equation $4\cos(\theta + 60°)\cos(\theta + 30°) = k$, determine the set of values of the constant k. [3]

15. June 2007 qu.7

(i) Sketch the graph of $y = \sec x$ for $0 \le x \le 2\pi$. [2]

(ii) Solve the equation $\sec x = 3$ for $0 \le x \le 2\pi$, giving the roots correct to 3 significant figures. [3]

(iii) Solve the equation $\sec \theta = 5 \operatorname{cosec} \theta$ for $0 \le \theta \le 2\pi$, giving the roots correct to 3s.f. [4]

16. June 2007 qu.9

(i) Prove the identity $\tan(\theta + 60°)\tan(\theta - 60°) \equiv \dfrac{\tan^2\theta - 3}{1 - 3\tan^2\theta}$. [4]

(ii) Solve, for $0° < \theta < 180°$, the equation $\tan(\theta + 60°)\tan(\theta - 60°) \equiv 4\sec^2\theta - 3$, giving your answers correct to the nearest 0.1°. [5]

(iii) Show that, for all values of the constant k, the equation $\tan(\theta + 60°)\tan(\theta - 60°) = k^2$ has two roots in the interval $0° < \theta < 180°$. [3]

17. Jan 2007 qu.2

It is given that θ is the acute angle such that $\sin\theta = \dfrac{12}{13}$. Find the exact value of

(i) $\cot\theta$, [2]

(ii) $\cos 2\theta$. [3]

18. Jan 2007 qu.5

(i) Express $4\cos\theta - \sin\theta$ in the form $R\cos(\theta + \alpha)$, where $R > 0$ and $0° < \alpha < 90°$. [3]

(ii) Hence solve the equation $4\cos\theta - \sin\theta = 2$, giving all solutions for which $-180° < \theta < 180°$. [5]

19. June 2006 qu.5

(i) Write down the identity expressing $\sin 2\theta$ in terms of $\sin\theta$ and $\cos\theta$. [1]

(ii) Given that $\sin\alpha = \dfrac{1}{4}$ and α is acute, show that $\sin 2\alpha = \frac{1}{8}\sqrt{15}$. [3]

(iii) Solve, for $0° < \beta < 90°$, the equation $5\sin 2\beta \sec\beta = 3$. [3]

20. June 2006 qu.8

(i) Express $5\cos x + 12\sin x$ in the form $R\cos(x - \alpha)$, where $R > 0$ and $0° < \alpha < 90°$. [3]

(ii) Hence give details of a pair of transformations which transforms the curve $y = \cos x$ to the curve $y = 5\cos x + 12\sin x$. [3]

(iii) Solve, for $0° < x < 360°$, the equation $5\cos x + 12\sin x = 2$, giving your answers correct to the nearest 0.1°. [5]

21. Jan 2006 qu.2

Solve, for $0° < \theta < 360°$, the equation $\sec^2\theta = 4\tan\theta - 2$. [5]

22. Jan 2006 qu.9

(i) By first writing $\sin 3\theta$ as $\sin(2\theta + \theta)$, show that $\sin 3\theta = 3\sin\theta - 4\sin^3\theta$. [4]

(ii) Determine the greatest possible value of $9\sin\left(\frac{10}{3}\alpha\right) - 12\sin^3\left(\frac{10}{3}\alpha\right)$,

and find the smallest positive value of α (in degrees) for which that greatest value occurs. [3]

(iii) Solve, for $0° < \beta < 90°$, the equation $3\sin 6\beta \operatorname{cosec} 2\beta = 4$. [6]

23. June 2005 qu.5

(i) Express $3\sin\theta + 2\cos\theta$ in the form $R\sin(\theta + \alpha)$, where $R > 0$ and $0° < \alpha < 90°$. [3]

(ii) Hence solve the equation $3\sin\theta + 2\cos\theta = \frac{7}{2}$, giving all solutions for which

$0° < \theta < 360°$. [5]

24. June 2005 qu.7

(i) Write down the formula for $\cos 2x$ in terms of $\cos x$. [1]

(ii) Prove the identity $\frac{4\cos 2x}{1+\cos 2x} = 4 - 2\sec^2 x$. [3]

(iii) Solve, for $0 < x < 2\pi$, the equation $\frac{4\cos 2x}{1+\cos 2x} = 3\tan x - 7$. [5]